

Analytical solution of problem about moderately strong evaporation (condensation) for one-dimensional kinetic equation

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Abstract

For one-dimensional linear kinetic equations analytical solutions of problems about moderately strong evaporation (condensation), when frequency of collisions of molecules is constant, are received. The equation and distribution function are linearize concerning the absolute Maxwellian, given far from a wall. Quantities of of temperature and concentration jumps are found. Distributions of concentration, mass velocity and temperature are constructed.

Key words: kinetic equation, collisional frequency, moderately strong evaporation (condensation), analytical solution, distribution of macroparameters.

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Введение

History of this problem in linear statement and with application one-dimensional collisional integral is stated in works [1, 2]. In these works attempts of the exact solution of problem about "strong" liquid evaporation in vacuum have been undertaken. Thus linearization was spent concerning equilibrium distribution function, given far from an evaporation surface. It has allowed to consider influence of movement of

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gas from a wall on behaviour of gas in Knudsen layer, remaining thus in frameworks linear approach. Velocity of the expiration of gas (and others parametres) were included into distribution function by nonlinear method. It is possible to name the similar approach "quasilinear". Despite the model solution and problem, it allows to describe correctly a number of the basic qualitative characteristics of evaporation. Then concerns first of all separation Mach number equal to unit ($\mathbf{M} = 1$).

In work [1] for the solution of problem the method of resolvent has been used. And in [2, 3] a method of boundary value problem from theory of functions of complex variable has been used. In [4] with attraction of methods of the functional analysis was resolvability of a problem in one interval ($0 < U < \sqrt{3/2}$) is shown and unsolvability in other $U > \sqrt{3/2}$, U is the dimensionless velocity of evaporation. In work [5] approximation F_N -method has been used. In the monography [6] (Chapter III, §4) this problem was studied in abstract statement.

Most nearly to the exact solution of this problem in a case of evaporation Siewert and Thomas have approached [2], and in a case condensation — Cercignani and Frezzotti [3]. However in these works [2, 3] authors could not receive exact relations between quantities of temperature jump, jump of density and velocity of evaporation (concentration).

The analytical solution of this problem has been finished in our work [8].

In works [10]–[13] the linear one-dimensional kinetic equation with collisional integral of BGK (Bhatnagar, Gross and Krook) and frequency of collisions, affine depending on the module of molecular velocity was considered.

The considered physical problem consists in the solution of the boundary problems for the modelling kinetic equation. Required physical quantities contain in boundary conditions. Substitution (Case' ansatz)

$$h_\eta(x, \mu) = \exp\left(-\frac{x}{\eta + U}\right)\Phi(\eta, \mu)$$

at once reduces the kinetic equation to the characteristic equation. From the solution of the characteristic equation are found eigen functions in space of generalized functions.

Further the structure of discrete and continuous spectra of the characteristic equations is investigated. Further the theorem about expansion of solution of boundary problem on generalized eigen functions is proved. The proof is reduced to the solution of singular integral equations with Cauchy kernel. This equation is reduced to the solution of the Riemann boundary value problem on semiaxis $[-U, +\infty)$. After the solution of corresponding homogeneous boundary value problem is found the general solution of a non-homogeneous boundary value problem. Unknown coefficients (physical quantities) of solution expansion from conditions of resolvability of boundary value problem are found.

1. Statement of problem and basic equations

Let us consider evaporation (condensation) of a liquid from the flat surfaces $x = 0$ in the vacuum occupying half-space $x > 0$. We take one-dimensional BGK-equation

$$\zeta \frac{\partial f}{\partial x} = \nu [\Phi(x, \zeta) - f(x, \zeta)], \quad (1.1)$$

where $f(x, \zeta)$ is the distribution function, ζ is the molecular velocity in the axes direction x , ν is the collisional frequency, $\Phi(x, \zeta)$ is the local Maxwellian,

$$\Phi(x, \zeta) = \frac{\rho(x)}{\sqrt{2\pi RT(x)}} \exp \left\{ -\frac{[\zeta - v(x)]^2}{2RT(x)} \right\}.$$

Here

$$\rho(x) = \int_{-\infty}^{\infty} f(x, \zeta) d\zeta,$$

$$v(x) = \frac{1}{\rho(x)} \int_{-\infty}^{\infty} \zeta f(x, \zeta) d\zeta,$$

$$T(x) = \frac{1}{R\rho(x)} \int_{-\infty}^{\infty} [\zeta - v(x)]^2 f(x, \zeta) d\zeta.$$

Jumps of temperature and density are required to be found

$$\varepsilon_T = \frac{T_s - T_\infty}{T_\infty}, \quad \varepsilon_\rho = \frac{\rho_s - \rho_\infty}{\rho_\infty},$$

where T_s , ρ_s are temperature and density of gas directly nearby at a wall.

Also density distribution $\rho(x)$, mass velocity $v(x)$ and temperatures at $x > 0$ are required to be found.

We believe that molecules are reflected purely diffusively from a wall. It means, that molecules are reflected from a wall with Maxwell distribution, i.e.

$$f(x = 0, \zeta) = f_s(\zeta), \quad \zeta > 0,$$

where

$$f_s(\zeta) = \frac{\rho_s}{\sqrt{2\pi RT_s}} \exp\left(-\frac{\zeta^2}{2RT_s}\right).$$

Let us assume, that far from a surface steam condition is described by equilibrium distribution characterized by the constant velocity of evaporation (condensation) v_∞ , density ρ_∞ and temperature T_∞ , i.e.

$$\Phi(\infty, \zeta) \equiv f_\infty(\zeta) = \frac{\rho_\infty}{\sqrt{2\pi RT_\infty}} \exp\left\{-\frac{[\zeta - v_\infty]^2}{2RT_\infty}\right\}.$$

Following [1], we will be linearize function of distribution and local Maxwellian concerning $f_\infty(\zeta)$. Entering the shift variable $c = \zeta - v_\infty$, we will write

$$f(x, \zeta) = f_\infty(c)[1 + h(x, c)], \quad (1.2)$$

where

$$f_\infty(\zeta) = \frac{\rho_\infty}{\sqrt{2\pi RT_\infty}} \exp\left\{-\frac{c^2}{2RT_\infty}\right\}.$$

We put in linear approximation

$$\rho(x) = \rho_\infty + \delta\rho(x),$$

$$T(x) = T_\infty + \delta T(x),$$

$$v(x) = v_\infty + \delta v(x).$$

Let us pass to dimensionless variables: to dimensionless coordinate x_1 , dimensionless molecular speed μ , dimensionless mass velocity of gas $U(x)$

$$x_1 = \frac{\nu x}{\sqrt{2RT_\infty}}, \quad \mu = \frac{c}{\sqrt{2RT_\infty}}, \quad U(x) = \frac{v(x)}{\sqrt{2RT_\infty}}.$$

Further dimensionless coordinate x_1 we will designate again through x .

Let us enter also the dimensionless given velocity of evaporation (condensation)

$$U = U_\infty = U(\infty) = \frac{v_\infty}{\sqrt{2RT_\infty}}.$$

By means of these designations we receive, that in linear approach

$$\begin{aligned} \Phi(x, \zeta) - f(x, \zeta) &= \\ &= f_\infty(\mu) \left[\frac{\delta \rho(x)}{\rho_\infty} + 2\mu \delta U(x) + \left(\mu^2 - \frac{1}{2} \right) \frac{\delta T(x)}{T_\infty} - h(x, \mu) \right]. \end{aligned}$$

Let us consider distributions of density, mass velocity and temperature also we will express their relative changes with help of function $h(x, \zeta)$.

For density we have

$$\frac{\rho(x)}{\rho_\infty} = 1 + \frac{\delta \rho(x)}{\rho_\infty}, \quad \frac{\delta \rho(x)}{\rho_\infty} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2} h(x, \mu) d\mu. \quad (1.3)$$

For mass velocity we have

$$U(x) = U + \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2} \mu h(x, \mu) d\mu. \quad (1.4)$$

For temperature distribution we have

$$T(x) = \frac{1}{R\rho(x)} \int_{-\infty}^{\infty} [c - \delta v(x)]^2 f_\infty(c) [1 + h(x, c)] dc =$$

$$= \frac{1}{R\rho(x)} \int_{-\infty}^{\infty} c^2 f_{\infty}(c) dc + \frac{1}{R\rho(x)} \int_{-\infty}^{\infty} c^2 f_{\infty}(c) h(x, c) dc.$$

From here, passing to dimensionless velocity of integration, we receive

$$\frac{T(x)}{T_{\infty}} = \frac{\rho_{\infty}}{\rho(x)} + 2 \frac{\rho_{\infty}}{\rho(x)} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2} \mu^2 h(x, \mu) d\mu.$$

Noticing that

$$\frac{\rho_{\infty}}{\rho(x)} = 1 - \frac{\delta\rho(x)}{\rho_{\infty}},$$

From the previous we receive

$$\frac{T(x)}{T_{\infty}} = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2} \left(\mu^2 - \frac{1}{2} \right) h(x, \mu) d\mu. \quad (1.5)$$

Now we can definitively formulate the one-dimensional linear kinetic equation with integral of collisions in the form of BGK

$$(\mu + U) \frac{\partial h}{\partial x} + h(x, \mu) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} q(\mu, \mu') h(x, \mu') d\mu'. \quad (1.6)$$

Here $q(\mu, \mu')$ is the kernel (or indicatrix) of equation,

$$q(\mu, \mu') = 1 + 2\mu\mu' + 2\left(\mu^2 - \frac{1}{2}\right)\left(\mu'^2 - \frac{1}{2}\right).$$

Let us understand with boundary conditions. As distribution function far from a wall passes in Maxwell distribution function, given far from a wall

$$\lim_{x \rightarrow +\infty} f(x, \zeta) = f_{\infty}(\zeta),$$

from here follows, that for function $h(x, \zeta)$ at once follows boundary condition far from a wall

$$h(+\infty, \mu) = 0. \quad (1.7)$$

The condition diffusion reflexion of molecules from a wall means, that

$$f_{\infty}(c)[1 + h(0, c)] = f_s(c), \quad c > -v_{\infty}.$$

From here we receive, that

$$h(0, c) = \frac{\rho_s}{\rho_\infty} \sqrt{\frac{T_\infty}{T_s}} \exp \left[-\frac{(c + v_\infty)^2}{2RT_s} + \frac{c^2}{2RT_\infty} \right]$$

In linear approximation we have

$$-\frac{(c + v_\infty)^2}{2RT_s} + \frac{c^2}{2RT_\infty} = -\frac{(\mu + U)^2}{1 + \varepsilon_T} + \mu^2 = -2U\mu + \varepsilon_T\mu^2.$$

Hence, from here we receive the second boundary condition

$$h(0, \mu) = \varepsilon_\rho - 2U\mu + \varepsilon_T\left(\mu^2 - \frac{1}{2}\right), \quad \mu > -U. \quad (1.8)$$

So, the boundary problem about moderately strong evaporation (condensation) for one-dimensional gas consists in finding of such solution of the equation (1.6) which satisfies to boundary conditions (1.7) and (1.8).

2. Separation of variables. Dispersion function. General solution of kinetic equation

Substitution (ansatz of Case)

$$h_\eta(x, \mu) = \exp\left(-\frac{x}{U + \eta}\right) \Phi(\eta, \mu) \quad (2.1)$$

at once reduces the equation (1.6) to the characteristic

$$\begin{aligned} & (\eta - \mu)\Phi(\eta, \mu) = \\ & = \frac{1}{\sqrt{\pi}}(\eta + U) \left[n_{(0)}(\eta) + 2\mu n_1(\eta) + 2\left(\mu^2 - \frac{1}{2}\right) \left(n_2(\eta) - \frac{1}{2}n_0(\eta) \right) \right]. \end{aligned} \quad (2.2)$$

Here the designation is entered

$$n_k(\eta) = \int_{-\infty}^{\infty} e^{-\mu^2} \Phi(\eta, \mu) \mu^k d\mu, \quad k = 0, 1, 2.$$

Multiplying the equation (1.6) on $\mu^k e^{-\mu^2}$ ($k = 0, 1$) and integrating on all real axis, we receive two equations

$$n_1(\eta) = -Un_0(\eta)$$

and

$$n_2(\eta) = -Un_1(\eta) = U^2n_0(\eta).$$

By means of two last equalities the equation (2.2) will be rewritten in the form

$$(\eta - \mu)\Phi(\eta, \mu) = \frac{1}{\sqrt{\pi}}(\eta + U)q(-U, \mu)n_0(\eta), \quad (2.3)$$

where

$$q(-U, \mu) = 1 - 2U\mu + 2\left(U^2 - \frac{1}{2}\right)\left(\mu^2 - \frac{1}{2}\right).$$

Further we will accept the following condition of normalization

$$n_0(\eta) \equiv \int_{-\infty}^{\infty} e^{-\mu^2} \Phi(\eta, \mu) d\mu \equiv 1. \quad (2.4)$$

Now the characteristic equation

$$(\eta - \mu)\Phi(\eta, \mu) = \frac{1}{\sqrt{\pi}}(\eta + U)q(-U, \mu)$$

has in space of generalized functions the following solution

$$\Phi(\eta, \mu) = \frac{1}{\sqrt{\pi}}(\eta + U)q(-U, \mu)P\frac{1}{\eta - \mu} + g(\eta)\delta(\eta - \mu), \quad (2.5)$$

where

$$\eta, \mu \in (-\infty, +\infty).$$

Here Px^{-1} means distribution — a principal value of integral at integration of expression x^{-1} , $\delta(x)$ is the Dirac delta-function.

Substituting expression (2.5) in the condition of normalization (2.4), we find, that

$$g(\eta) = e^{\eta^2} \lambda(\eta),$$

where $\lambda(z)$ is the dispersion function,

$$\lambda(z) = 1 + \frac{z + U}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2} \frac{q(-U, \mu)}{\mu - z} d\mu. \quad (2.6)$$

Thus, eigen functions of the characteristic equation, corresponding to continuous spectrum, look like

$$\Phi(\eta, \mu) = \frac{1}{\sqrt{\pi}}(\eta + U)q(-U, \mu)P\frac{1}{\eta - \mu} + e^{\eta^2}\lambda(\eta)\delta(\eta - \mu). \quad (2.5')$$

According to (2.1), eigen solution of equation (1.6) decreasing far from a wall and corresponding to continuous spectrum, look like

$$h_\eta(x, \mu) = \exp\left(-\frac{x}{\eta + U}\right)\Phi(\eta, \mu), \quad \eta > -U, \quad -\infty < \mu < +\infty.$$

Expression (2.6) for dispersion function it is possible to express through dispersion function of plasma

$$\lambda_C(z) = 1 + \frac{z}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\mu^2} d\mu}{\mu - z} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\mu e^{-\mu^2} d\mu}{\mu - z}.$$

Substituting an explicit form of function $q(-U, \mu)$ in (2.6), we receive, that

$$\lambda(z) = 1 + (z + U) \left\{ \left[2\left(U^2 - \frac{1}{2}\right)z - 2U \right] \lambda_C(z) + \left(\frac{3}{2} - U^2\right)t(z) \right\}.$$

According to formulas of Sokhotsky, for boundary values dispersion function we have

$$\lambda^\pm(\mu) = \lambda(\mu) + i\sqrt{\pi}(\mu + U)e^{-\mu^2}q(-U, \mu), \quad -\infty < \mu < +\infty.$$

Here

$$\lambda(\mu) = 1 + \frac{\mu + U}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} \frac{q(-U, \mu')}{\mu' - \mu} d\mu',$$

and integral in this equality is understood as singular in sense of principal value on Cauchy.

From formulas of Sokhotsky we receive, that

$$\begin{aligned} \lambda^+(\mu) - \lambda^-(\mu) &= 2\sqrt{\pi}i(\mu + U)e^{-\mu^2}q(-U, \mu), \\ \frac{\lambda^+(\mu) + \lambda^-(\mu)}{2} &= \lambda(\mu), \quad -\infty < \mu < +\infty. \end{aligned}$$

By means of an argument principle [15] it is possible to show, that dispersion function has no complex zero in the finite parts of complex plane.

Let us expand dispersion function in Laurent series in a vicinity infinitely remote point

$$\lambda(z) = -U\left(U^2 - \frac{3}{2}\right)\frac{1}{z^3} - \frac{3}{2}\left(U^2 - \frac{1}{2}\right)\frac{1}{z^4} - 3U\left(U^2 - \frac{3}{2}\right)\frac{1}{z^5} + \dots, \quad z \rightarrow \infty. \quad (2.7)$$

From expansion (2.7) it is visible, that infinitely remote point is zero of the third order, if $U \neq 0$ and $U^2 \neq \frac{3}{2}$, and zero of the fourth order otherwise, i.e. if $U = 0$ or $U^2 = \frac{3}{2}$. This last case was investigated in work [9].

To infinitely remote point, as to a multiple point of the discrete spectrum, there correspond following discrete modes (solutions)

$$h_0(x, \mu) = 1,$$

$$h_1(x, \mu) = \mu,$$

$$h_2(x, \mu) = \mu^2,$$

and

$$h_3(x, \mu) = \left(\mu^2 - \frac{3}{2}\right)(x - \text{sign } \mu),$$

if $U = 0$ or $U^2 = \frac{3}{2}$.

3. Analytical solution of the problem about moderately strong evaporation (condensation)

Having eigen solutions corresponding to continuous and discrete spectra, we construct the general solution of the equation (1.6) in the form of integral on the continuous spectrum and the linear combination of

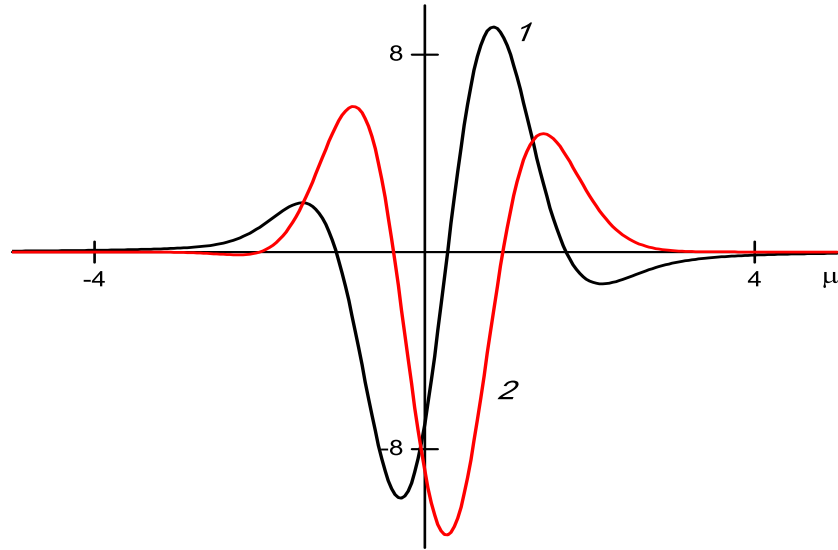


Fig. 1. Real (curve 1) and imaginary (curve 2) parts of dispersion function, $U = 2$.

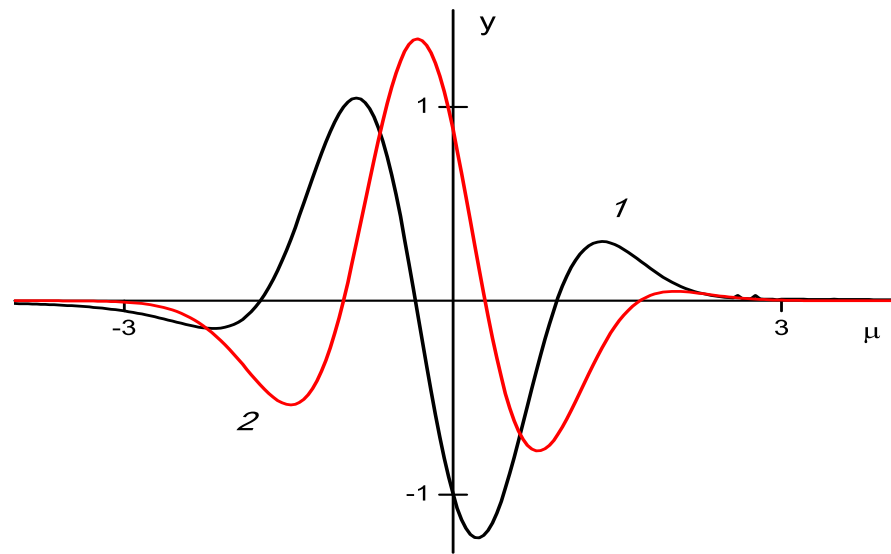


Fig. 2. Real (curve 1) and imaginary (curve 2) parts of dispersion function, $U = 1$.

discrete eigen solutions

$$h(x, \mu) = A_0 + A_1\mu + A_2\mu^2 + \int_{-\infty}^{+\infty} e^{-x/(\eta+U)} \Phi(\eta, \mu) a(\eta) d\eta. \quad (3.1)$$

Constants A_0, A_1, A_2 and function $a(\eta)$ are coefficients of expansion (3.1) corresponding accordingly to discrete and continuous spectra. These coefficients are defined from boundary conditions (1.7) and (1.8).

Substituting expansion (3.1) in the boundary condition (1.7), we receive, that $A_0 = A_1 = A_2 = 0$ and $a(\eta) = 0$ at $\eta < -U$. Hence, expansion (3.1) becomes simpler

$$h(x, \mu) = \int_{-U}^{+\infty} e^{-x/(\eta+U)} \Phi(\eta, \mu) a(\eta) d\eta. \quad (3.2)$$

Let us substitute now expansion (3.2) in the boundary condition (1.8). We receive the following integral equation

$$\varepsilon_\rho - 2U\mu + \varepsilon_T \left(\mu^2 - \frac{1}{2} \right) = \int_{-U}^{+\infty} \Phi(\eta, \mu) a(\eta) d\eta, \quad \mu > -U. \quad (3.3)$$

Substituting in (3.3) eigen functions of the continuous spectrum (2.5'), we come to the singular integral equation with Cauchy kernel

$$h(0, \mu) = q(-U, \mu) \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} \frac{(\eta + U) a(\eta)}{\eta - \mu} d\eta + e^{\mu^2} \lambda(\mu) a(\mu), \quad (3.4)$$

where $\mu > -U$,

$$h(0, \mu) = \varepsilon_\rho - 2U\mu + \varepsilon_T \left(\mu^2 - \frac{1}{2} \right).$$

Let us enter auxiliary function

$$N(z) = \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} \frac{(\eta + U) a(\eta)}{\eta - z} d\eta. \quad (3.5)$$

This function is analytic in the complex plane \mathbb{C} with cut $\mathbb{R}_U = [-U, +\infty]$. Its boundary values from above and from below on the cut satisfy to formulas of Sokhotsky

$$\begin{aligned} N^+(\mu) - N^-(\mu) &= 2\sqrt{\pi}i(\eta + U)a(\eta), \\ \frac{N^+(\mu) + N^-(\mu)}{2} &= N(\mu), \end{aligned}$$

where

$$N(\mu) = \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} \frac{(\eta + U)a(\eta)}{\eta - \mu} d\eta,$$

and last integral is understood in sense of a principal value.

By means of boundary values of auxiliary function $N(z)$ and dispersion function $\lambda(z)$ we will reduce the singular integral equation (3.4) to nonhomogeneous boundary value problem

$$\begin{aligned} \lambda^+(\mu)[q(-U, \mu)N^+(\mu) - h(0, \mu)] &= \\ = \lambda^-(\mu)[q(-U, \mu)N^-(\mu) - h(0, \mu)], \quad \mu > -U. \end{aligned} \quad (3.6)$$

For the solution of a nonhomogeneous boundary value problem (3.6) we will solve at first corresponding homogeneous Riemann boundary value problem

$$\frac{X^+(\mu)}{X^-(\mu)} = \frac{\lambda^+(\mu)}{\lambda^-(\mu)}, \quad \mu > -U. \quad (3.7)$$

In the problem (3.7) unknown function $X(z)$ is analytic in cut complex plane \mathbb{C} with cut $\mathbb{R}_U = [-U, +\infty]$. The solution of the problem (3.7) essentially depends on quantity and sign of parametre U , from which dispersion function depends and which defines semiaxis on which the boundaty value problem is given.

We notice that

$$|\lambda^+(\mu)| = |\lambda^-(\mu)|, \quad \lambda^+(\mu) = \overline{\lambda^-(\mu)}, \quad -\infty < \mu < +\infty.$$

Let us enter the angle $\theta(\mu) = \arg \lambda^+(\mu)$ is the principal value of argument fixed in the point $\mu = -U$ by the condition $\theta(-U) = 0$ (see

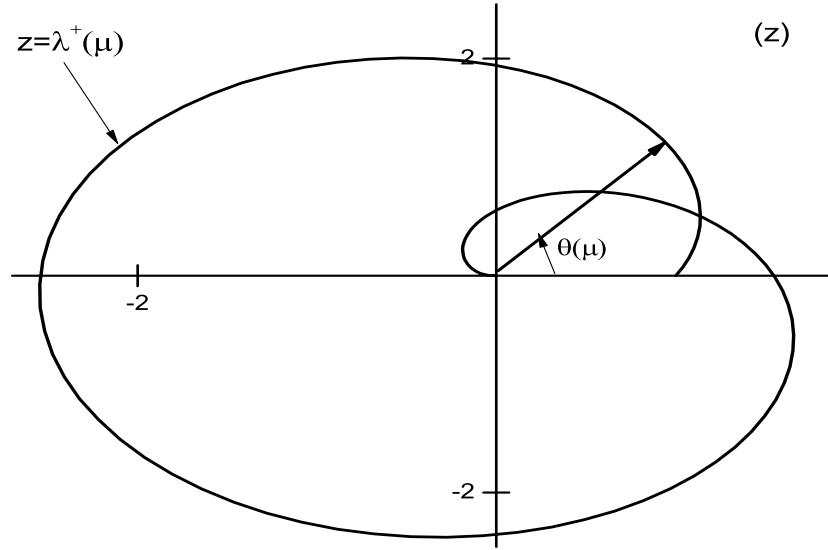


Fig. 3. The curve $z = \lambda^+(\mu)$, $-(\sqrt{3/2} + 0.1) \leq \mu \leq +\infty$, the case $U = \sqrt{3/2} + 0.1$. Incrementation of angle $\theta(\mu)$ equals 3π on semiaxis $-(\sqrt{3/2} + 0.1) \leq \mu \leq +\infty$.

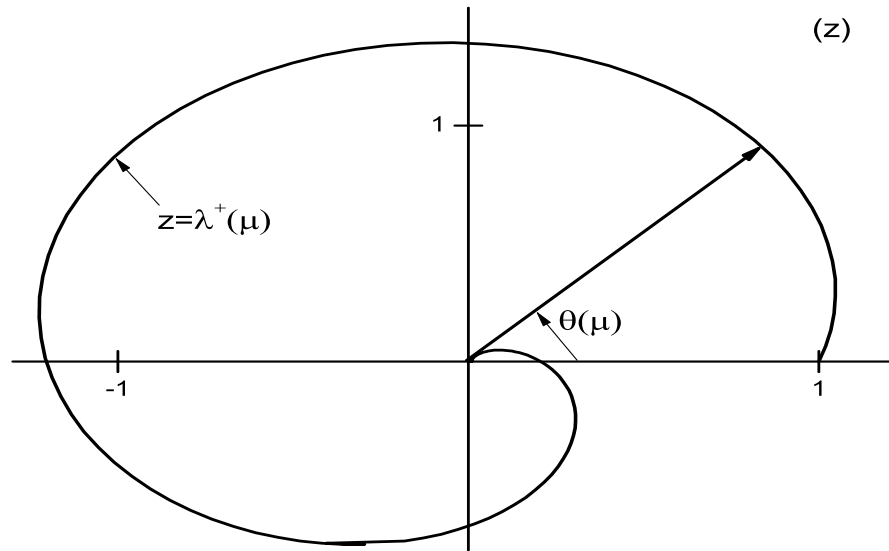


Fig. 4. The curve $z = \lambda^+(\mu)$, $-1 \leq \mu \leq +\infty$, the case $U = 1$. Incrementation of angle $\theta(\mu)$ equals 2π on semiaxis $-1 \leq \mu \leq +\infty$.

Figs. 3 and 4). It is easy to see, that coefficient of the boundary value problem (3.7)

$$G(\mu) = \frac{\lambda^+(\mu)}{\lambda^-(\mu)}$$

is equal

$$G(\mu) = \frac{\lambda^+(\mu)}{\lambda^-(\mu)} = \frac{\lambda(\mu) + is(\mu)}{\lambda(\mu) - is(\mu)} = e^{2i\theta(\mu)}.$$

Here

$$\begin{aligned} s(\mu) &= \sqrt{\pi}e^{-\mu^2}(\mu + U)q(-U, \mu), \\ \lambda(\mu) &= 1 + (\mu + U) \left\{ \lambda_C(\mu) \left[2 \left(U^2 - \frac{1}{2} \right) \mu - 2U \right] - t(\mu) \left(U^2 - \frac{3}{2} \right) \right\}, \\ \lambda_C(\mu) &= 1 - 2\mu^2 e^{-\mu^2} \int_0^1 e^{\mu\tau^2} d\tau, \\ t(\mu) &= -2\mu e^{-\mu^2} \int_0^1 e^{\mu\tau^2} d\tau. \end{aligned}$$

We notice that

$$q(-U, -U) = 2 \left(U^2 - \frac{1}{2} \right)^2 + 2U^2 + 1 > 0, \quad q(-U, 0) = - \left(U^2 - \frac{3}{2} \right).$$

The function $q(-U, \mu)$ has two real roots

$$\mu_{1,2}^q = \frac{U \pm \sqrt{D(U)}}{2 \left(U^2 - \frac{1}{2} \right)},$$

where

$$D(U) = 2 \left(U^2 - \frac{3}{4} \right)^2 + \frac{3}{8} > 0.$$

We notice that

$$\lim_{U \rightarrow \pm\infty} \mu_1^q(U) = \frac{1}{\sqrt{2}},$$

$$\lim_{U \rightarrow \pm\infty} \mu_2^q(U) = -\frac{1}{\sqrt{2}}.$$

Let us consider family of curves $\Gamma_U : z = G(t), -U \leq \mu \leq +\infty$. These curves are closed: they begin and come to end in one point $z = 1$.

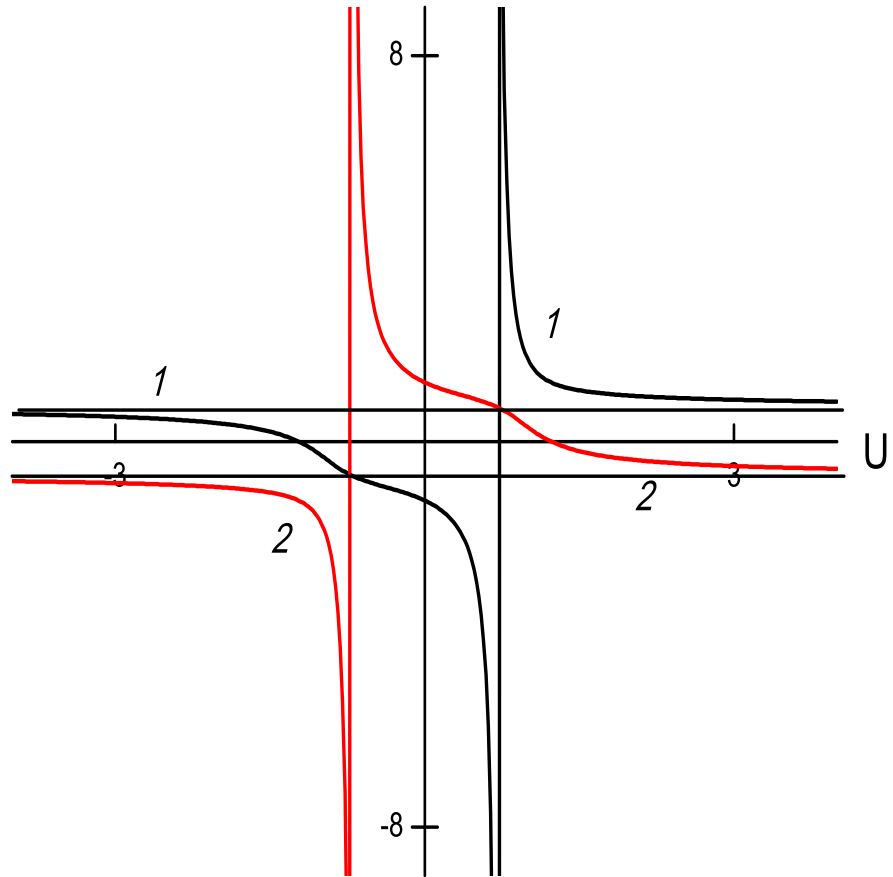


Fig. 5. Behavior of roots $\mu_1(U)$ (curve 1) and $\mu_2(U)$ (curve 2), $U = \pm \frac{1}{\sqrt{2}}$ are vertical asymptotics, $y = \pm \frac{1}{\sqrt{2}}$ are horizontal asymptotics.

Really, it is easy to see, that

$$\lambda(-U) = \lambda(+\infty) = 1, \quad s(-U) = s(+\infty) = 0.$$

For the solution of homogeneous Riemann problem (3.7) important calculate index of coefficient of problem $G(t)$ on the closed semiaxis \mathbb{R}_U , i.e. number of turns concerning of origin of coordinates, which the curve Γ_U runs when the parametre t describes semiaxis \mathbb{R}_U . As $\arg G(t) = 2\theta(t)$, from here follows, that the number of turns of the curve Γ_U is equal to the doubled number of turns of the angle $\theta(t)$ on the semiaxis \mathbb{R}_U , i.e. it is equal to the angle increment $\theta(t)$ on the semiaxis \mathbb{R}_U , divided on 2π . So for finding angle increments $\theta(t)$ we will consider family of curves

$$\gamma_U : z = \lambda^+(\mu) = \lambda(\mu) + is(\mu), \quad -U \leq \mu \leq +\infty.$$

At first we will consider the case of moderately strong evaporation ($U > 0$).

Without the proof we will inform, that in the case $U > \sqrt{3/2}$ the angle increment $\theta(t)$ on the semiaxis \mathbb{R}_U is equal 3π (see Fig. 3). It means, the index of problem is equal to three: $\varkappa(G) = 3\pi$. In case of $0 < U < \sqrt{3/2}$ the angle increment $\theta(t)$ on the semiaxis \mathbb{R}_U is equal 2π (see Fig. 4). It means, the index of problem is equal to two: $\varkappa(G) = 2\pi$.

Homogeneous Riemann problem (3.7) we will reduce to problem of definition of analytical function on quantity of its jump on the cut

$$\ln X^+(\mu) - \ln X^-(\mu) = 2i[\theta(\mu) + k\pi], \quad -U \leq \mu \leq +\infty, \quad (3.8)$$

where $k = 0, \pm 1, \pm 2, \dots$.

In the case $U > \sqrt{3/2}$ in (3.8) it is necessary to take $k = -3$, i.e. we consider the problem

$$\ln X^+(\mu) - \ln X^-(\mu) = 2i[\theta(\mu) - 3\pi], \quad -U \leq \mu \leq +\infty, \quad (3.8')$$

As the solution of problem (3.8') we take the limited in the point $z = -U$ function

$$X(z) = \frac{1}{(z + U)^3} e^{V(z)}, \quad (3.9)$$

where

$$V(z) = \frac{1}{\pi} \int_{-U}^{\infty} \frac{\theta(\mu) - 3\pi}{\mu - z} d\mu.$$

By means of homogeneous Riemann boundary value problem (3.7) we will transform nonhomogeneous problem (3.6) to the problem of definition of the analytical functions on its jump on the cut

$$\begin{aligned} X^+(\mu)[q(-U, \mu)N^+(\mu) - h(0, \mu)] = \\ = X^-(\mu)[q(-U, \mu)N^-(\mu) - h(0, \mu)], \quad \mu > -U. \end{aligned} \quad (3.10)$$

Considering behaviour of the functions entering into boundary value condition (3.10), we receive, that the problem (3.10) has only trivial solution

$$X(z)[q(-U, z)N(z) - h(0, z)] \equiv 0,$$

whence we find

$$N(z) = \frac{h(0, z)}{q(-U, z)}.$$

However, this solution cannot be accepted as function $N(z)$, entered by equality (3.5). This function is limited into infinitely remote point, while the auxiliary function (3.5) vanishes in infinitely remote point.

Thus, the considered boundary problem has no solution in the case $U > \sqrt{3/2}$.

In case of $0 < U < \sqrt{3/2}$ in (3.8) it is necessary to take $k = -2$, i.e. to consider the problem

$$\ln X^+(\mu) - \ln X^-(\mu) = 2i[\theta(\mu) - 2\pi], \quad -U \leq \mu \leq +\infty, \quad (3.8'')$$

As the solution of problem (3.8'') we take the limited in the point $z = -U$ function

$$X(z) = \frac{1}{(z + U)^2} e^{V(z)}, \quad (3.10)$$

where

$$V(z) = \frac{1}{\pi} \int_{-U}^{\infty} \frac{\theta(\mu) - 2\pi}{\mu - z} d\mu.$$

Let us notice, that the angle $\theta(\mu)$ (see Fig. 6) conveniently to calculate under the formula

$$\theta(\mu) = \operatorname{arccctg} \frac{\lambda(\mu)}{s(\mu)} + \begin{cases} 0 & -U \leq \mu \leq \mu_1^q(U), \\ \pi & \mu_1^q(U) \leq \mu \leq \mu_2^q(U), \\ 2\pi & \mu_2^q(U) \leq \mu \leq +\infty. \end{cases}$$

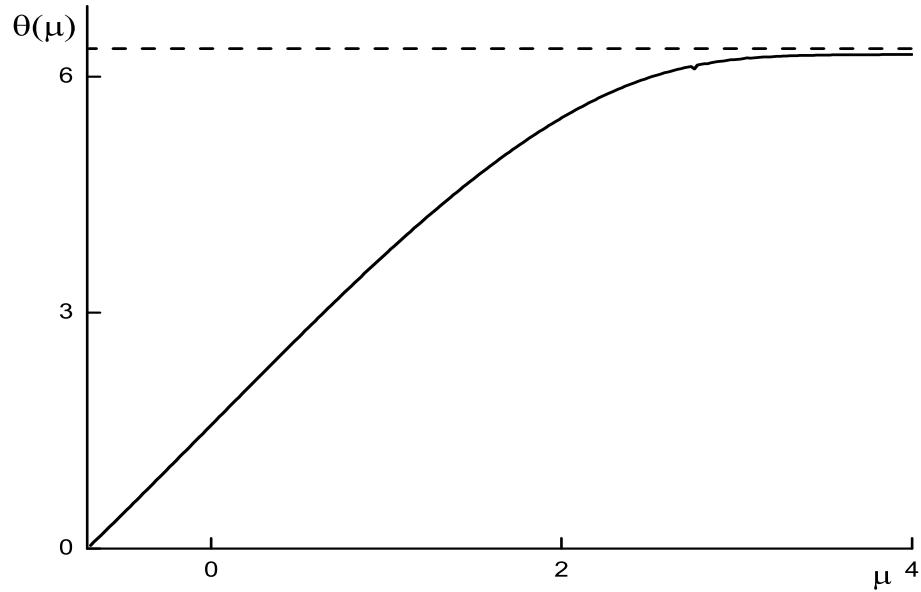


Fig. 6. The angle $\theta(\mu)$ in case $U = 1/\sqrt{2}$. Incrementation of angle on semiaxis $[-1/\sqrt{2}, +\infty]$ equals 2π .

In considered case the boundary value problem (3.10) has the solution

$$X(z)[q(-U, z)N(z) - h(0, z)] = C_0,$$

where C_0 is the arbitrary constant.

From this solution we find auxiliary function $N(z)$

$$N(z) = \frac{h(0, z) + \frac{C_0}{X(z)}}{q(-U, z)}. \quad (3.11)$$

The solution (3.11) represents meromorphic function. Its denominator is function $q(-U, z)$. This function has in the case $U \neq \frac{1}{\sqrt{2}}$ two real zero $\mu_1^q(U)$ and $\mu_2^q(U)$, and in the case $U = \frac{1}{\sqrt{2}}$ function $q(-U, z) = 1 - \sqrt{2}z$ has unique zero. To this case we will return later.

Poles in points $\mu_1^q(U)$ and $\mu_2^q(U)$ are destroyed by conditions

$$h(0, \mu_\alpha^q) + \frac{C_0}{X(\mu_\alpha^q)} = 0, \quad \alpha = 1, 2. \quad (3.12)$$

Condition of vanishing of functions $N(z)$ in infinitely remote point is reached by the condition

$$C_0 = -\varepsilon_T. \quad (3.13)$$

Equality (3.13) follows from expansion in the Laurent series on to negative degrees of z the right part of equality (3.11).

Let us notice, that as function $N(z)$ is defined in the complex plane, at its limiting values $N^\pm(\mu)$ from above and from below in points $\mu_1^q(U)$ and $\mu_2^q(U)$ also exist simple poles. That them to destroy, we will demand performance four equalities

$$h(0, \mu_\alpha^q) - \frac{\varepsilon_T}{X^\pm(\mu_\alpha)} = 0, \quad \alpha = 1, 2. \quad (3.14)$$

Let us show, that these equalities coincide with equalities (3.12), i.e. are carried out automatically. Really, we will result without a conclusion integral representation of function $X(z)$

$$X(z) = \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} \frac{e^{-\mu^2}(\mu + U)q(-U, \mu)X^+(\mu)}{\lambda^+(\mu)(\mu - z)} d\mu.$$

From integral representation it is visible, that, as density this integral in points $\mu_1^q(U)$ and $\mu_2^q(U)$ equals to zero, boundary values of this

integral from above and from below in points $\mu_1^q(U)$ and $\mu_2^q(U)$ coincide with values singular integral in these points. Thus,

$$X^\pm(\mu_\alpha^q) = X(\mu_\alpha^q) + i\sqrt{\pi}e^{-\mu^2}(\mu + U)q(-U, \mu_\alpha^q)\frac{X^+(\mu)}{\lambda^+(\mu)} = X(\mu_\alpha^q),$$

because $q(-U, \mu_\alpha^q) = 0$, $\alpha = 1, 2$.

From the equations (3.12) and (3.13) we find required quantities of temperature jump and density jump as functions of velocity of evaporation U

$$\varepsilon_T = 2U \frac{(\mu_1^q - \mu_2^q)X(\mu_1^q)X(\mu_2^q)}{(\mu_1^{q2} - \mu_2^{q2})X(\mu_1^q)X(\mu_2^q) + X(\mu_1^q) - X(\mu_2^q)} \quad (3.15)$$

and

$$\varepsilon_\rho = 2U\mu_1^q - \left[\mu_1^{q2} - \frac{1}{2} - \frac{1}{X(\mu_1^q)} \right] \varepsilon_T. \quad (3.16)$$

Let us return to the case $U = \frac{1}{\sqrt{2}}$. The solution (3.11) let us present in the explicit form

$$N(z) = \frac{C_0(z + U)e^{-V(z)} + \varepsilon_\rho - 2Uz + \varepsilon_T(z^2 - 1/2)}{1 - \sqrt{2}z}.$$

The pole is eliminated by one condition

$$2C_0e^{-V(U)} + \varepsilon_\rho - 1 = 0,$$

whence we find

$$C_0 = \frac{1 - \varepsilon_\rho}{2} e^{V(U)}.$$

Destroying the pole of the second order in infinitely remote point, we receive

$$\varepsilon_T = -C_0 = -\frac{1 - \varepsilon_\rho}{2} e^{V(U)},$$

and

$$C_0(-V_1 + \sqrt{2}) - 2U = 0,$$

where

$$V_1 = -\frac{1}{\pi} \int_{-U}^{\infty} [\theta(\tau) - 2\pi] d\tau.$$

Expansion in vicinity infinitely remote point has been thus used

$$e^{-V(z)} = 1 - \frac{V_1}{z} + \dots, \quad z \rightarrow \infty.$$

From last equations we find quantities of jump of temperature and density jump

$$\begin{aligned} \varepsilon_T &= -\frac{\sqrt{2}}{\sqrt{2} - V_1} = \frac{1}{V_1 - \sqrt{2}}(2U), \\ \varepsilon_\rho &= 1 - \frac{2\sqrt{2}e^{-V(U)}}{\sqrt{2} - V_1} = -\frac{\sqrt{2}(1 - 2e^{-V(U)}) - V_1}{\sqrt{2}(V_1 - \sqrt{2})}(2U). \end{aligned}$$

Let us consider now various cases of condensation.

Let at first $-\sqrt{3/2} < U < 0$. The analysis shows, that in this case argument increment $\theta(t)$ on the semiaxis $\mathbb{R}_U = [-U, +\infty]$ is equal π . Therefore as the solution of the homogeneous Riemann problem we take limited in the point $z = -U$ function

$$X(z) = \frac{1}{z + U}e^{V(z)},$$

where

$$V(z) = \frac{1}{\pi} \int_{-U}^{\infty} \frac{\theta(\mu) - \pi}{\mu - z} d\mu.$$

The general solution of the problem (3.11) looks like now

$$N(z) = \frac{\frac{C_0 + C_1 z}{X(z)} + h(0, z)}{q(-U, z)}. \quad (3.17)$$

From the condition of vanishing of function $N(z)$ in infinitely remote point, we find that

$$C_1 = -\varepsilon_T.$$

From the condition of elimination of poles in points μ_1^q and μ_2 we receive two equations

$$X(\mu_\alpha)h(0, \mu_\alpha) + C_0 + C_1\mu_\alpha = 0, \quad \alpha = 1, 2.$$

From this system we find

$$C_0 = \frac{1}{2} \left\{ \varepsilon_T \left[\mu_1 + \mu_2 - X(\mu_1) \left(\mu_1^2 - \frac{1}{2} \right) - X(\mu_2) \left(\mu_2^2 - \frac{1}{2} \right) \right] + \right. \\ \left. + 2U [\mu_1 X(\mu_1) + \mu_2 X(\mu_2)] - \varepsilon_\rho [X(\mu_1) + X(\mu_2)] \right\}$$

and

$$\varepsilon_T = 2U \frac{\mu_1 X(\mu_1) - \mu_2 X(\mu_2)}{\mu_2 - \mu_1 + X(\mu_1)(\mu_1 - 1/2) - X(\mu_2)(\mu_2 - 1/2)} \\ - \varepsilon_\rho \frac{X(\mu_1) - X(\mu_2)}{\mu_2 - \mu_1 + X(\mu_1)(\mu_1 - 1/2) - X(\mu_2)(\mu_2 - 1/2)}.$$

This solution is ambiguous. It contains the free unknown parametre ε_ρ .

Let now $U < -\sqrt{3/2}$. In this case it is possible to show, that increment of angle $\theta(\mu)$ on the semiaxis $\mathbb{R}_U = [-U, +\infty]$ is equal to zero. Therefore as the solution of the homogeneous Riemann problem we take limited in the point $z = -U$ function

$$X(z) = e^{V(z)},$$

where

$$V(z) = \frac{1}{\pi} \int_{-U}^{\infty} \frac{\theta(\mu) d\mu}{\mu - z}.$$

Hence, the general solution of a nonhomogeneous boundary value problem (3.10) looks like

$$N(z) = \frac{\frac{C_0 + C_1 z + C_2 z^2}{X(z)} + h(0, z)}{q(-U, z)},$$

where C_0, C_1, C_2 are arbitrary constans.

From condition of vanishing of solution in infinitely remote point we find

$$C_2 = -\varepsilon_T.$$

From condition of elimination of poles we have the following equalities

$$C_0 + C_1\mu_\alpha^q + C_2\mu_\alpha^{q^2} + h(0, \mu_\alpha^q)X(\mu_\alpha^q) = 0, \quad \alpha = 1, 2.$$

From this system we find

$$C_0 = \varepsilon_T(\mu_1^q + \mu_2^q) - \frac{X(\mu_1)h(0, \mu_1^q) - X(\mu_2)h(0, \mu_2^q)}{\mu_1^q - \mu_2^q}$$

and

$$C_0 = \frac{1}{2} \left\{ -C_1(\mu_1^q + \mu_2^q) + \varepsilon_T(\mu_1^{q^2} + \mu_2^{q^2}) - X(\mu_1^q)h(0, \mu_1^q) - X(\mu_2^q)h(0, \mu_2^q) \right\}.$$

This solution, as well as previous, ambiguously. It contains two free parameters ε_T и ε_ρ .

For the unequivocal solution of the problem on condensation it is necessary to set three parameters U , ε_T и ε_ρ .

4. Temperature jump and weak evaporation (condensation). Distribution of gas macroparameters

Quantities of jumps of temperature and density we will present in the following form

$$\varepsilon_T = K(U)(2U), \quad \varepsilon_\rho = R(U)(2U),$$

where coefficients of jumps of temperature and density $K(U)$ and $R(U)$ are given by formulas

$$K(U) = \frac{(\mu_1^q - \mu_2^q)X(\mu_1^q)X(\mu_2^q)}{(\mu_1^{q^2} - \mu_2^{q^2})X(\mu_1^q)X(\mu_2^q) + X(\mu_1^q) - X(\mu_2^q)}$$

and

$$R(U) = \mu_1^q - \left[\mu_1^{q^2} - \frac{1}{2} - \frac{1}{X(\mu_1^q)} \right] K(U).$$

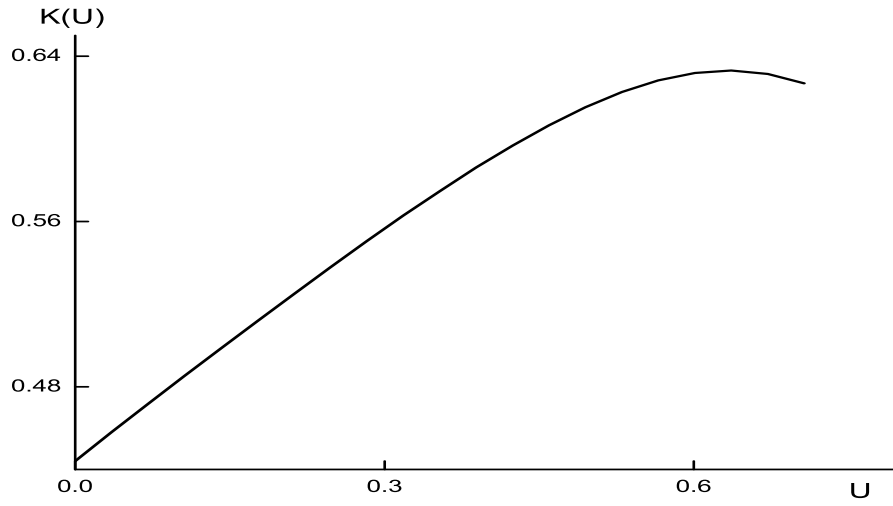


Fig. 7. Dependence of temperature jump coefficient on dimensionless quantity of evaporation velocity in the range $0 \leq U \leq 1/\sqrt{2}$.

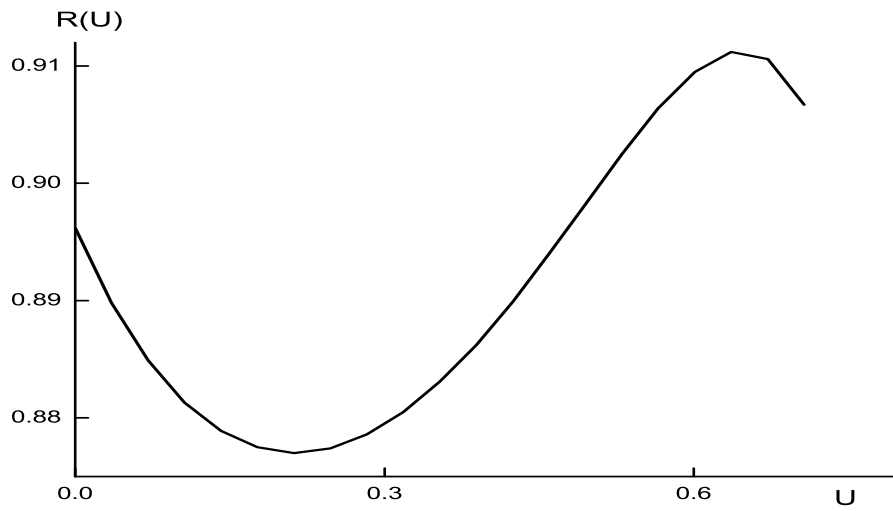


Fig. 8. Dependence of density jump coefficient on dimensionless quantity of evaporation velocity in the range $0 \leq U \leq 1/\sqrt{2}$.

Let us carry out numerical calculations in the case $U = \frac{1}{\sqrt{2}}$. In this case $V_1 = 3.0095$, $V(U) = 1.8376$. Therefore

$$\varepsilon_T = 0.6268(2U_0) = 0.8864, \quad \varepsilon_\rho = -1.7612(2U_0) = -2.4907.$$

For comparison we will present results on weak evaporation with application of the one-dimensional kinetic equation with collisional frequency, proportional to the module of molecular velocity

$$\varepsilon_T = -0.5046(2U), \quad \varepsilon_n = -0.2523(2U).$$

For comparison we will present coefficients of jump of temperature and jump of concentration found by means of the one-dimensional kinetic equations with constant frequency of collisions [12]

$$\varepsilon_T = -0.4443(2U), \quad \varepsilon_n = -0.8958(2U).$$

Coefficients of continuous spectrum we will find from Sokhotsky' formulas for auxiliary function and the constructed solution

$$N^+(\mu) - N^-(\mu) = 2\sqrt{\pi}i(\mu + U)a(\mu) = -\frac{\varepsilon_T}{q(-U, \mu)} \left[\frac{1}{X^+(\mu)} - \frac{1}{X^-(\mu)} \right].$$

It is easy to see, that

$$\frac{1}{X^+(\mu)} - \frac{1}{X^-(\mu)} = -2i \frac{\sin \theta(\mu)}{X(\mu)}.$$

Hence, the factor of the continuous spectrum is equal

$$(\mu + U)a(\mu) = \frac{\sin \theta(\mu)}{\sqrt{\pi}q(-U, \mu)X(\mu)} \varepsilon_T. \quad (4.1)$$

Substituting the found expression in expansion (3.2), we find discontinuous in the point $\mu = -U$ at $x = 0$ distribution function

$$\frac{h(x, \mu)}{\varepsilon_T} = \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} \exp\left(-\frac{x}{\eta + U}\right) \frac{\sin \theta(\eta)}{X(\eta)} d\eta +$$

$$+ \exp\left(\mu^2 - \frac{x}{\mu + U}\right) \frac{\lambda(\mu) \sin \theta(\mu)}{\sqrt{\pi} q(-U, \mu) X(\mu)} \Theta_+(\mu + U).$$

Here $\Theta_+(x)$ is the known Heaviside function (unit jump at origin of coordinates).

Substituting coefficient of the continuous spectrum (4.1) in distribution of gas density (1.3), and considering, that normalization of eigen function of zero order is identically equal to unit, we receive

$$\begin{aligned} \frac{\rho(x)}{\rho_\infty} &= 1 + \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} e^{-x/(\eta+U)} d\eta \int_{-\infty}^{\infty} e^{-\mu^2} \Phi(\eta, \mu) d\mu = \\ &= 1 + \frac{\varepsilon_T}{\pi} \int_{-U}^{\infty} e^{-x/(\eta+U)} \frac{\sin \theta(\eta) d\eta}{(\eta + U) q(-U, \eta) X(\eta)}. \end{aligned}$$

Let us substitute (4.1) in distribution of mass velocity (1.4) and considering, that normalization of the first order is equal $-U$, we receive

$$\begin{aligned} U(x) &= U + \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} e^{-x/(\eta+U)} d\eta \int_{-\infty}^{\infty} e^{-\mu^2} \mu \Phi(\eta, \mu) d\mu = \\ &= U \left[1 - \frac{\varepsilon_T}{\pi} \int_{-U}^{\infty} e^{-x/(\eta+U)} \frac{\sin \theta(\eta) d\eta}{(\eta + U) q(-U, \eta) X(\eta)} \right]. \end{aligned}$$

Substituting (4.1) in distribution of temperature (1.5) and considering, that normalization of the second order is equal U^2 , we receive

$$\begin{aligned} \frac{T(x)}{T_\infty} &= 1 + \frac{2}{\sqrt{\pi}} \int_{-U}^{\infty} e^{-x/(\eta+U)} a(\eta) d\eta \int_{-\infty}^{\infty} e^{-\mu^2} \left(\mu^2 - \frac{1}{2} \right) \Phi(\eta, \mu) d\mu = \\ &= 1 + 2 \left(U^2 - \frac{1}{2} \right) \frac{\varepsilon_T}{\pi} \int_{-U}^{\infty} e^{-x/(\eta+U)} \frac{\sin \theta(\eta) d\eta}{(\eta + U) q(-U, \eta) X(\eta)}. \end{aligned}$$

From last formulas it is visible, that distribution of mass velocity and density are connected by equality

$$\frac{\rho(x)}{\rho_\infty} + \frac{U(x)}{U_\infty} = 2,$$

and density and temperature distribution are connected by equality

$$\frac{T(x)}{T_\infty} = 1 + 2\left(U^2 - \frac{1}{2}\right) \left[\frac{\rho(x)}{\rho_\infty} - 1\right].$$

5. Conclusion

In the present work the analytical solution of boundary problem about moderately strong evaporation (condensation) with application of the one-dimensional kinetic equation with constant frequency of collisions of molecules is considered.

The carried out analysis shows, that both with physical, and with mathematical point of view of a problem of evaporation and condensation are non-symmetrical.

Let us notice, that for one-dimensional gas value of the Mach number $\mathbf{M} = 1$ corresponds to velocity of evaporation (condensation) $U = \sqrt{3/2}$.

From results of work follows, that this quantity renders essential influence on evaporation and condensation modes.

The modes of condensation established in work at the various values of velocity of condensation $U < 0$ (subsonic and supersonic cases) correspond to results of numerical calculations, done in work [7].

Function of distribution of gas molecules in explicit form is received, and also distributions of density of gas, its mass velocity and temperature in half-space $x > 0$.

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